EUREF Technical Note 1: Relationship and Transformation between the International and the European Terrestrial Reference Systems

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1. Introduction

The European Terrestrial Reference System 89 (ETRS89) was adopted in 1990 in Firenze, Italy, following the EUREF Resolution 1 that states: "The IAG Subcommision for the European Reference Frame recommends that the system to be adopted by EUREF will be coincident with the ITRS at the epoch 1989.0 and fixed to the stable part of the Eurasian Plate and will be known as European Terrestrial Reference System 89 (ETRS89)".

Following this resolution, a detailed description of ETRS89 and its relationship with the International Terrestrial Reference System (ITRS), including transformation formula were published in Boucher and Altamimi (1992).

The purpose of this Technical Note is to (1) recall the mathematical relationship linking ETRS89 to the ITRS, (2) provide the users with all the necessary information allowing to transform station positions and velocities from any ITRS realization to any ETRS89 realization, designated hereafter by ITRF$_{yy}$ and ETRF$_{yy}$, respectively, and (3) recommend a procedure to follow when realizing the ETRS89 at the national or regional level using GNSS data and the International GNSS Service (IGS) products. This Technical Note summarizes and replaces the old memo of Boucher and Altamimi that was first published in September 20, 1993 and last updated in May 18, 2011.

2. ITRS and ETRS89 Relationship

The currently adopted approach to realize the ITRS is to consider its associated frame, the ITRF, as a secular (linear) frame, where the 7 components of its defining parameters (origin, scale and orientation) are specified at a given epoch t_0 and their time evolutions are considered as varying linearly with time.

The general relationship between two systems (A and B) allowing to transform station positions and velocities from system A to system B is given by:

$$\left\{ \begin{array}{lcl} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_B & = & \begin{pmatrix} x \\ y \\ z \end{pmatrix}_A + T + D \begin{pmatrix} x \\ y \\ z \end{pmatrix}_A + R \begin{pmatrix} x \\ y \\ z \end{pmatrix}_A \\ \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}_B & = & \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}_A + \dot{T} + \dot{D} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_A + \dot{R} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_A \end{array} \right. \tag{1}$$

T is the translation vector, $T = (T_x, T_y, T_z)^T$, D is the scale factor and R is the matrix containing the rotation angles, given by (following the IERS Conventions (Petit and Luzum, 2010, Ch. 4)):

$$R = \begin{pmatrix} 0 & -R_z & R_y \\ R_z & 0 & -R_x \\ -R_y & R_x & 0 \end{pmatrix}$$

The dotted parameters designate their time derivatives.

The ETRS89 definition, as specified by the Firenze Resolution cited above, and detailed in Boucher and Altamimi (1992) implies the following two conditions:

1. The ETRS89 coincides with the ITRS at epoch 1989.0. This condition leads to consider that the 7 transformation parameters between ITRS and ETRS89 are all zeros at epoch 1989.0.

2. The ETRS89 is fixed to the stable part of the Eurasian tectonic plate. This condition implies that the ETRS89 is co-moving with the Eurasian tectonic plate, hence defining its time evolution. Therefore the time derivatives of the 7 parameters between ITRS and ETRS89 are zeros, except the three rotation rates. The three rotation rates are in fact the three components of the Eurasia angular velocity in the ITRF$_{yy}$ frames.

As a consequence of the above two conditions, it becomes straightforward to derive the transformation formulae allowing to link the ETRS89 to the ITRS, for both station positions and station velocities, using the following two equations:
For station positions at any epoch t:

$$X_{yy}^E(t) = X_{yy}^I(t) + T_{yy} + \begin{pmatrix} 0 & -\dot{R}3_{yy} & \dot{R}2_{yy} \\ \dot{R}3_{yy} & 0 & -\dot{R}1_{yy} \\ -\dot{R}2_{yy} & \dot{R}1_{yy} & 0 \end{pmatrix} \times X_{yy}^I(t).(t - 1989.0) \qquad (2)$$

and for station velocities:

$$\dot{X}_{yy}^E = \dot{X}_{yy}^I + \begin{pmatrix} 0 & -\dot{R}3_{yy} & \dot{R}2_{yy} \\ \dot{R}3_{yy} & 0 & -\dot{R}1_{yy} \\ -\dot{R}2_{yy} & \dot{R}1_{yy} & 0 \end{pmatrix} \times X_{yy}^I \qquad (3)$$

where $(X_{yy}^I, \dot{X}_{yy}^I)$ and $(X_{yy}^E, \dot{X}_{yy}^E)$ are the couples of station positions and velocities in the yy realizations of the two systems, ITRS and ETRS89, respectively.

The rotation rate parameters $\dot{R}1_{yy}, \dot{R}2_{yy}, \dot{R}3_{yy}$ are the three components of the Eurasia Euler vector (or angular velocity) expressed in the ITRF$_{yy}$. The translation vector T_{yy} is introduced in equation 2 to eventually account for global origin offset between the different ITRF versions.

It should be emphasized here that the ETRS89 definition does not explicitly nor implicitly specify any condition regarding station vertical motion (that can reach 1 cm/yr in Postglacial Rebound regions). Consequently station vertical velocities are the same in ITRS and ETRS89 corresponding frames.

It is also worth noting that the usage of equation 3 removes the global velocity of the Eurasia plate in the ITRF$_{yy}$ and leave some residual velocities that can however be large in deforming and seismic zones.

3. ETRS89 realizations derived from ITRF

Using equations 2 and 3, an ETRF$_{yy}$ can directly be derived from the corresponding ITRF$_{yy}$, by choosing 6 parameters : the three translation components of T_{yy} and the three rotation rates $\dot{R}1_{yy}, \dot{R}2_{yy}, \dot{R}3_{yy}$.

3.1. Translation Parameters T_{yy}

In order to satisfy the first ETRS89 condition (coincidence of ETRS89 and ITRS at epoch 1989.0), T_{yy} should normally be set to zero each time an ETRF$_{yy}$ is derived from an ITRF$_{yy}$. However in practice, this condition was satisfied so far only for the ETRF89 derivation which was considered as the initial ETRS89 realization. Subsequent T_{yy} were then estimated to account for origin component offsets between ITRF89 and successive ITRF solutions, as detailed in the memo of Boucher and Altamimi (2011).

After the release of ITRF2000, the subsequent ITRF$_{yy}$ origins tend toward a more stable origin. The estimated accuracy and stability of the ITRF2014 origin, as reflected by the level of agreement with the ITRF2008, are at the level of less than 3 mm at epoch 2010.0 and less than 0.2 mm/yr in time evolution (see Altamimi et al. (2016) for more details). Therefore an ETRF2014 is proposed where T_{yy} components are reset to zero, so that its origin coincides with that of ITRF2014.

3.2. Rotation Rate Parameters $\dot{R}_{yy}$

Up to the ETRF97, the Eurasia angular velocity was adopted from geophysical models which were used in the No-Net-Rotation condition implementation in the corresponding ITRF solutions: AM02 of Minster and Jordan (1978); NNR-NUVEL1 and NNR-NUVEL1-A of Argus and Gordon (1991), based on relative plate motion model of DeMets et al. (1990) and revised in 1994 (DeMets et al., 1994).

Starting with ITRF2000, angular velocities of the Eurasian plate, together with other plates, were estimated using ITRF velocity fields of ITRF2000, ITRF2005, ITRF2008 and ITRF2014, (Altamimi et al., 2002, 2007, 2012, 2017). The ETRF2000 was the first ETRS89 frame where the Eurasia angular velocity components were estimated using an ITRF velocity field (Altamimi and Boucher, 2002). The ITRF2014 Plate Motion Model (ITRF2014-PMM) published in Altamimi et al. (2017) is shown to be a more robust and precise model compared with past ITRF models, as it involves more sites (especially GNSS sites) and significantly more data. The uncertainty of the Eurasia angular velocity is 4 to 5 times smaller in ITRF2014-PMM than in ITRF2000 model. The determination of the ITRF2014-PMM follows strict criteria of site selection, satisfying the notion of rigid-plate motion hypothesis, at the level of (or better than) 0.3 mm/yr WRMS in average. The Eurasia Euler vector in particular involves 97 sites, with velocity residuals far less than 1 mm/yr, as depicted by Figure 3 of Altamimi et al. (2017).

3.3. Transformation Parameters from ITRF$_{yy}$ to ETRF$_{yy}$

Table 1 lists the sets of 14 transformation parameters from each ITRF$_{yy}$ to its corresponding ETRF$_{yy}$ published so far (see the old memo of Boucher and Altamimi (2011) for more details), including the newly proposed ETRF2014 where the three translation components are set to zero and the rotation rates (or Eurasia angular velocity components) are taken from Altamimi et al. (2017).

3.4. EUREF Recommendations

After the release of ITRF2005, the EUREF Governing Board (EUREF-GB), formerly Technical Working Group (TWG), recommended not to use the ETRF2005 and rather to adopt the ETRF2000 as a conventional frame of the ETRS89 system. This decision was taken by the TWG, noticing that coordinate shifts at epochs posterior to 1989.0 occur between ETRF$_{yy}$ frames which are originally due to equivalent shifts between the global ITRF frames. This is the example of coordinate shifts at epochs posterior to 1989.0 between ETRF2000 and ETRF2005. These shifts are due, mainly, to the Z-translation rate of 1.8 mm/yr between ITRF2000 and ITRF2005 as well as to the refined rotation rate values ($\dot{R}_{yy}$).

After the release of ITRF2014 (Altamimi et al., 2016), noting the improved accuracy and stability of the origin and scale parameters of the ITRF2014 and the significance of an improved ETRS89 realization based on the ITRF2014, the 2017 EUREF Resolution No. 1, urges the EUREF-GB to make available all the defining parameters of the ETRF2014 with an origin coinciding with that of ITRF2014 and to provide the full series of transformation parameters between ITRF and ETRF versions in an updated technical memo. The same resolution also recognizes the diverse requirements regarding national implementations of ETRS89, and respects the different countries' decisions on adopting their preferred ETRS89 realizations including the recommended ETRF2000.

The adoption of a particular ETRS89 realization by a country will depend on various criteria which are beyond the scope of this Technical Note. However, the adoption of ETRF2014 may be preferred for high precision applications and better consistency with the ITRF2014 precise geocentric origin, whereas ETRF2000 could be considered for geo-referencing purposes where agreement with realizations in neighboring countries is of concern. The coordinate differences between ETRF2014 and ETRF2000 may reach up to 7 cm.

4. ETRS89 realizations using GNSS Data

Procedure on how to express station coordinates of a GNSS network in the ITRF using the International GNSS Service (IGS) products can be found in Altamimi (2003). The old fashion of fixing or tightly constraining coordinates of a subset of stations to their ITRF values is no longer recommended. Instead, it is highly recommended to use the concept of minimum constraints to align a regional or national GNSS network solution to the ITRF, as described in Altamimi (2003) and in more details in Altamimi and Gross (2017).

4.1. Expression of a EUREF GNSS Network Solution in ETRS89

Detailed guidelines for EUREF densifications, GNSS data analysis and how to compute station coordinates in the ETRS89 can be found in Bruyninx et al. (2013).

For a given set of GNSS observations of a EUREF local or regional network, referred to a central epoch t_c, the recommended analysis procedure consists of two parts: (1) computing station coordinates in ITRS, followed by (2) transforming in ETRS89.

4.2. Processing GNSS Data in ITRS at epoch t_c

The following steps are recommended:

- use the most recent ITRF$_{yy}$ and the corresponding yy IGS products (Orbits, Clocks and Earth Orientation Parameters -EOPs);

- add to your network as many IGS/ITRF globally distributed stations as possible for an optimal alignement to the ITRF$_{yy}$;
- fix or constrain the orbits, clocks and EOPs to their IGS yy values;
- if needed, propagate the ITRF$_{yy}$ coordinates from their initial epoch t_0 to the central epoch t_c of the employed GNSS observations, using:

$$X^I_{yy}(t_c) = X^I_{yy}(t_0) + \dot{X}^I_{yy}.(t_c - t_0) \tag{4}$$

- use the minimum constraints approach to align the network solution to ITRF$_{yy}$ over the 7 reference frame parameters (origin, scale and orientation), applied to the ITRF/IGS stations, as described in Altamimi (2003) or Altamimi and Gross (2017). The minimum constraints option is believed to be implemented in most, if not all major scientific GNSS software packages;

The resulting station coordinates are then all consistent with ITRF$_{yy}$ at epoch t_c.

4.3. Transformation into ETRS89

There are two possible cases to transform station coordinates from ITRF$_{yy}$ to ETRF$_{yy}$ at epoch t_c:

Case A. GNSS data are processed in ITRF$_{yy}$ (e.g. ITRF2014) and the target ETRS89 frame is ETRF$_{yy}$ (i.e. ETRF2014). In this case equation 2 should be used where t is substituted by t_c, together with the values of T_{yy} and R_{yy} listed in Table 1.

Case B. GNSS data are processed in ITRF$_{yy}$ (e.g. ITRF2014) and the target ETRS89 frame is ETRF$_{xx}$ (e.g. ETRF93). In this case two-step procedure should be applied:

Step 1. Transform ITRF$_{yy}$ coordinates at t_c into ITRF$_{xx}$ using equation 1 and the IERS/ITRF published values. For convenience, the transformation parameters from ITRF2014 to all past ITRF solutions are listed in appendix A of this Technical Note. The transformation parameters between any two ITRF frames can easily be deduced from that table. Note that the values are provided at epoch 2010.0 and should be propagated at epoch t_c using equation 5, so that for any transformation parameter P, we have:

$$P(t_c) = P(2010.0) + \dot{P}.(t_c - 2010.0) \tag{5}$$

where $\dot{P}$ designates the rate of any one of the 7 parameters.

Step 2. This step is similar to **(Case A)** where equation 2 should be used to transform from ITRF$_{xx}$ to ETRF$_{xx}$.

The user should note that, in order to be fully compatible with the ETRS89 definition, it is not recommended to propagate the station coordinates from the central epoch of observations used, t_c, to any other epoch, such as 1989.0, by means of whatever intra plate velocities. However, countries in Postglacial Rebond regions or in deforming and seismic zones, may need to apply a deformation model to propagate coordinates of new determined points from epoch t_c to the reference epoch of their legal national reference frame.

The above two-step procedure could in fact be replaced by one-step procedure using 14 transformation parameters. This is the case, for example, of a direct transformation from any ITRF$_{yy}$ to ETRF2014 or ETRF2000, as described in the following subsection.

4.4. Transformation Parameters from ITRF$_{yy}$ to ETRF2014 and ETRF2000

Tables 2 and 3 list the 14 transformation parameters at epoch 2010.0 and their rates to be used when transforming from any ITRF$_{yy}$ into ETRF2014 and into ETRF2000, respectively. The transformation parameters in Tables 2 and 3 were computed by the summation of the transformation ITRF$_{yy}$-To-ITRF2014 and ITRF2014-To-ETRF2014 (and respectively for the ETRF2000, by replacing 2014 with 2000). The transformations ITRF2014-To-ETRF2014, and ITRF2000-To-ETRF2000 consist of the translation parameters and rotation rates which are taken from Table 1 of this Technical Note, while the rotation parameters at epoch 2010.0 are computed by multiplying the rotation rates by 21 (2010.0 - 1989.0).

The user should be aware that the transformation parameters listed in Tables 2 and 3 are expressed at epoch 2010.0. Since the transformation should be performed at the central epoch (t_c) of the observations used, then these transformation parameters should be propagated at epoch t_c, using equation 5. Therefore the 7 parameters propagated at epoch t_c should then be used to transform GNSS coordinates from ITRF$_{yy}$ to ETRF2014 or to ETRF2000.

The purpose of Tables 2 and 3 is to allow the users operating a direct transformation from any ITRF$_{yy}$ to ETRF2014 or ETRF2000, using the general transformation formula of equation 1.

It should be noted that the general two-step (or 14-parameter transformation) procedure could be applied to any other ETRF$_{yy}$ instead of ETRF2014 or ETRF2000. For instance, if a country has adopted ETRF93 and for various reasons wants to stick to that frame, then their GNSS station coordinates expressed in recent ITRF version (say ITRF2014) should first be transformed in ITRF93 and subsequently transformed in ETRF93 using the formula of this Technical Note.

4.5. Numerical Applications and Validation

When transforming their station coordinates from ITRS to ETRS89 at epoch t_c, the users are strongly advised to validate their obtained station coordinates in their preferred ETRS89 frame, by using the web-based tool available at the EUREF Permanent Network (EPN) web site:

http://epncb.oma.be/_productsservices/coord_trans/

Indeed, the tool allows transformation between any ITRS and ETRS89 frames, at any epoch.

For the user convenience, Appendix B of this Technical Note provides numerical applications of transformation from ITRF2014 to ETRF2014 and to ETRF2000, at two different epochs 2010.0 and 2020.0

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Table 1: Transformation parameters from ITRF$_{yy}$ to ETRFyy **at epoch 1989.0** and their rates/year

ETRF$_{yy}$	T1 mm	T2 mm	T3 mm	D 10^{-9}	R1 mas	R2 mas	R3 mas
ETRF2014	0.0	0.0	0.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.085	0.531	-0.770
ETRF2005	56.0	48.0	-37.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.054	0.518	-0.781
ETRF2000	54.0	51.0	-48.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.081	0.490	-0.792
ETRF97	41.0	41.0	-49.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.020	0.500	-0.650
ETRF96	41.0	41.0	-49.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.020	0.500	-0.650
ETRF94	41.0	41.0	-49.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.020	0.500	-0.650
ETRF93	19.0	53.0	-21.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.032	0.780	-0.670
ETRF92	38.0	40.0	-37.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.021	0.520	-0.680
ETRF91	21.0	25.0	-37.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.021	0.520	-0.680
ETRF90	19.0	28.0	-23.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.011	0.570	-0.710
ETRF89	0.0	0.0	0.0	0.00	0.000	0.000	0.000
rates	0.0	0.0	0.0	0.00	0.011	0.570	-0.710

Table 2: Transformation parameters from ITRF$_{yy}$ to ETRF2014 **at epoch 2010.0** and their rates/year

ITRF Solution	T1 mm	T2 mm	T3 mm	D 10^{-9}	R1 mas	R2 mas	R3 mas
ITRF2014	0.0	0.0	0.0	0.00	1.785	11.151	-16.170
rates	0.0	0.0	0.0	0.00	0.085	0.531	-0.770
ITRF2008	-1.6	-1.9	-2.4	0.02	1.785	11.151	-16.170
rates	0.0	0.0	0.1	-0.03	0.085	0.531	-0.770
ITRF2005	-2.6	-1.0	2.3	-0.92	1.785	11.151	-16.170
rates	-0.3	0.0	0.1	-0.03	0.085	0.531	-0.770
ITRF2000	-0.7	-1.2	26.1	-2.12	1.785	11.151	-16.170
rates	-0.1	-0.1	1.9	-0.11	0.085	0.531	-0.770
ITRF97	-7.4	0.5	62.8	-3.80	1.785	11.151	-16.430
rates	-0.1	0.5	3.3	-0.12	0.085	0.531	-0.790
ITRF96	-7.4	0.5	62.8	-3.80	1.785	11.151	-16.430
rates	-0.1	0.5	3.3	-0.12	0.085	0.531	-0.790
ITRF94	-7.4	0.5	62.8	-3.80	1.785	11.151	-16.430
rates	-0.1	0.5	3.3	-0.12	0.085	0.531	-0.790
ITRF93	50.4	-3.3	60.2	-4.29	4.595	14.531	-16.570
rates	2.8	0.1	2.5	-0.12	0.195	0.721	-0.840
ITRF92	-15.4	-1.5	70.8	-3.09	1.785	11.151	-16.430
rates	-0.1	0.5	3.3	-0.12	0.085	0.531	-0.790
ITRF91	-27.4	-15.5	76.8	-4.49	1.785	11.151	-16.430
rates	-0.1	0.5	3.3	-0.12	0.085	0.531	-0.790
ITRF90	-25.4	-11.5	92.8	-4.79	1.785	11.151	-16.430
rates	-0.1	0.5	3.3	-0.12	0.085	0.531	-0.790
ITRF89	-30.4	-35.5	130.8	-8.19	1.785	11.151	-16.430
rates	-0.1	0.5	3.3	-0.12	0.085	0.531	-0.790

Table 3: Transformation parameters from ITRF$_{yy}$ to ETRF2000 **at epoch 2010.0** and their rates/year

ITRF Solution	T1 mm	T2 mm	T3 mm	D 10^{-9}	R1 mas	R2 mas	R3 mas
ITRF2014	54.7	52.2	-74.1	2.12	1.701	10.290	-16.632
rates	0.1	0.1	-1.9	0.11	0.081	0.490	-0.792
ITRF2008	53.1	50.3	-76.5	2.14	1.701	10.290	-16.632
rates	0.1	0.1	-1.8	0.08	0.081	0.490	-0.792
ITRF2005	52.1	51.2	-71.8	1.20	1.701	10.290	-16.632
rates	-0.2	0.1	-1.8	0.08	0.081	0.490	-0.792
ITRF2000	54.0	51.0	-48.0	0.00	1.701	10.290	-16.632
rates	0.0	0.0	0.0	0.00	0.081	0.490	-0.792
ITRF97	47.3	52.7	-11.3	-1.68	1.701	10.290	-16.892
rates	0.0	0.6	1.4	-0.01	0.081	0.490	-0.812
ITRF96	47.3	52.7	-11.3	-1.68	1.701	10.290	-16.892
rates	0.0	0.6	1.4	-0.01	0.081	0.490	-0.812
ITRF94	47.3	52.7	-11.3	-1.68	1.701	10.290	-16.892
rates	0.0	0.6	1.4	-0.01	0.081	0.490	-0.812
ITRF93	105.1	48.9	-13.9	-2.17	4.511	13.670	-17.032
rates	2.9	0.2	0.6	-0.01	0.191	0.680	-0.862
ITRF92	39.3	50.7	-3.3	-0.97	1.701	10.290	-16.892
rates	0.0	0.6	1.4	-0.01	0.081	0.490	-0.812
ITRF91	27.3	36.7	2.7	-2.37	1.701	10.290	-16.892
rates	0.0	0.6	1.4	-0.01	0.081	0.490	-0.812
ITRF90	29.3	40.7	18.7	-2.67	1.701	10.290	-16.892
rates	0.0	0.6	1.4	-0.01	0.081	0.490	-0.812
ITRF89	24.3	16.7	56.7	-6.07	1.701	10.290	-16.892
rates	0.0	0.6	1.4	-0.01	0.081	0.490	-0.812

Appendix A

Transformation Parameters at epoch **2010.0** and their rates from ITRF2014 to past ITRF$_{yy}$

ITRF Solution	T1 mm	T2 mm	T3 mm	D 10^{-9}	R1 mas	R2 mas	R3 mas
ITRF2008	1.6	1.9	2.4	-0.02	0.00	0.00	0.00
rates	0.0	0.0	-0.1	0.03	0.00	0.00	0.00
ITRF2005	2.6	1.0	-2.3	0.92	0.00	0.00	0.00
rates	0.3	0.0	-0.1	0.03	0.00	0.00	0.00
ITRF2000	0.7	1.2	-26.1	2.12	0.00	0.00	0.00
rates	0.1	0.1	-1.9	0.11	0.00	0.00	0.00
ITRF97	7.4	-0.5	-62.8	3.80	0.00	0.00	0.26
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02
ITRF96	7.4	-0.5	-62.8	3.80	0.00	0.00	0.26
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02
ITRF94	7.4	-0.5	-62.8	3.80	0.00	0.00	0.26
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02
ITRF93	-50.4	3.3	-60.2	4.29	-2.81	-3.38	0.40
rates	-2.8	-0.1	-2.5	0.12	-0.11	-0.19	0.07
ITRF92	15.4	1.5	-70.8	3.09	0.00	0.00	0.26
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02
ITRF91	27.4	15.5	-76.8	4.49	0.00	0.00	0.26
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02
ITRF90	25.4	11.5	-92.8	4.79	0.00	0.00	0.26
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02
ITRF89	30.4	35.5	-130.8	8.19	0.00	0.00	0.26
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02
ITRF88	25.4	-0.5	-154.8	11.29	0.10	0.00	0.26
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02

Appendix B: Numerical Applications of Transformation

In the numerical examples below, we select the following station position components (XYZ) and velocity components ($\dot{X}\dot{Y}\dot{Z}$) expressed in ITRF2014 (in meter and meter per year):

Example 1: at epoch t_c = 2010.0

```
4027893.6719   307045.9064  4919475.1704 (ITRF2014, Epoch 2010.0)
     -.01361       0.01676       0.01044
```

Case A: Transformation from ITRF2014 to ETRF2014:
Using equation 2 (for station positions) and equation 3 (for station velocities) and the values of T_{yy} and $\dot{R}_{yy}$ from Table 1 gives:

```
4027893.9620   307045.5480  4919474.9553 (ETRF2014, Epoch 2010.0)
     0.00020       -.00030       0.00020
```

Case B: Transformation from ITRF2014 to ETRF2000:
Step 1: Transformation in ITRF2000, using the general transformation formula given by equation 1 and the 14 transformation parameters from the table of Appendix A, we obtain:

```
4027893.6812   307045.9082  4919475.1547 (ITRF2000, Epoch 2010.0)
     -.01307       0.01690       0.00908
```

Step 2: Transformation from ITRF2000 to ETRF2000 using equation 2 for station positions and equation 3 for velocities, T_{yy} and $\dot{R}_{yy}$ from ITRF2000 to ETRF2000 listed in Table 1:

```
4027894.0053   307045.5939  4919474.9083 (ETRF2000, Epoch 2010.0)
     -.00020       -.00050       -.00036
```

The above station positions and velocities can be obtained by one-step procedure using equation 1 and the 14 transformation parameters from ITRF2014 to ETRF2000 listed in Table 3.

Example 2: at epoch t_c = 2020.0, omitting station velocities which are the same as in Example 1.

```
4027893.5358   307046.0740  4919475.2748 (ITRF2014, Epoch 2020.0)
```

Case A: Transformation from ITRF2014 to ETRF2014, in a similar way as in Example 1:

```
4027893.9639   307045.5450  4919474.9573 (ETRF2014, Epoch 2020.0)
```

Case B: Transformation from ITRF2014 to ETRF2000:
Step 1: Transformation in ITRF2000, using equation 1 and the 14 transformation parameters from the table of Appendix A, propagated at epoch 2020.0 using equation 5, we obtain:

```
4027893.5505   307046.0772  4919475.2456 (ITRF2000, Epoch 2020.0)
```

Step 2: Transformation of the above station positions from ITRF2000 to ETRF2000 using equation 2, T_{yy} and $\dot{R}_{yy}$ from ITRF2000 to ETRF2000 listed in Table 1:

```
4027894.0033   307045.5889  4919474.9047 (ETRF2000, Epoch 2020.0)
```